

# Connectivity analysis of electrophysiological data



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# M/EEG signal characteristics considered during analysis

timecourse of activity

-> ERP

spectral characteristics

-> power spectrum

temporal changes in power

-> time-frequency response (TFR)

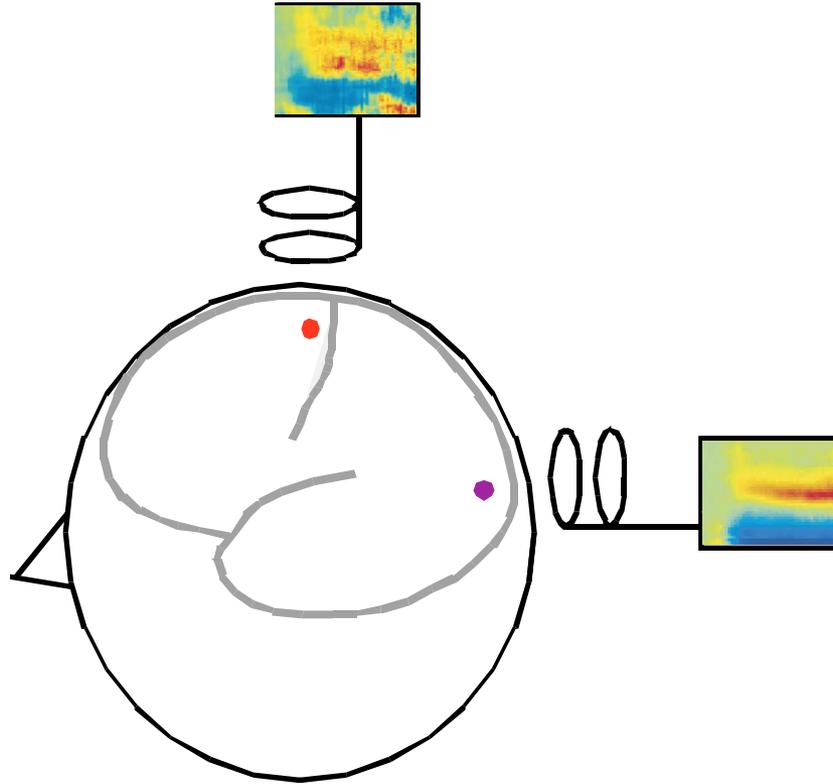
spatial distribution of activity

-> source reconstruction

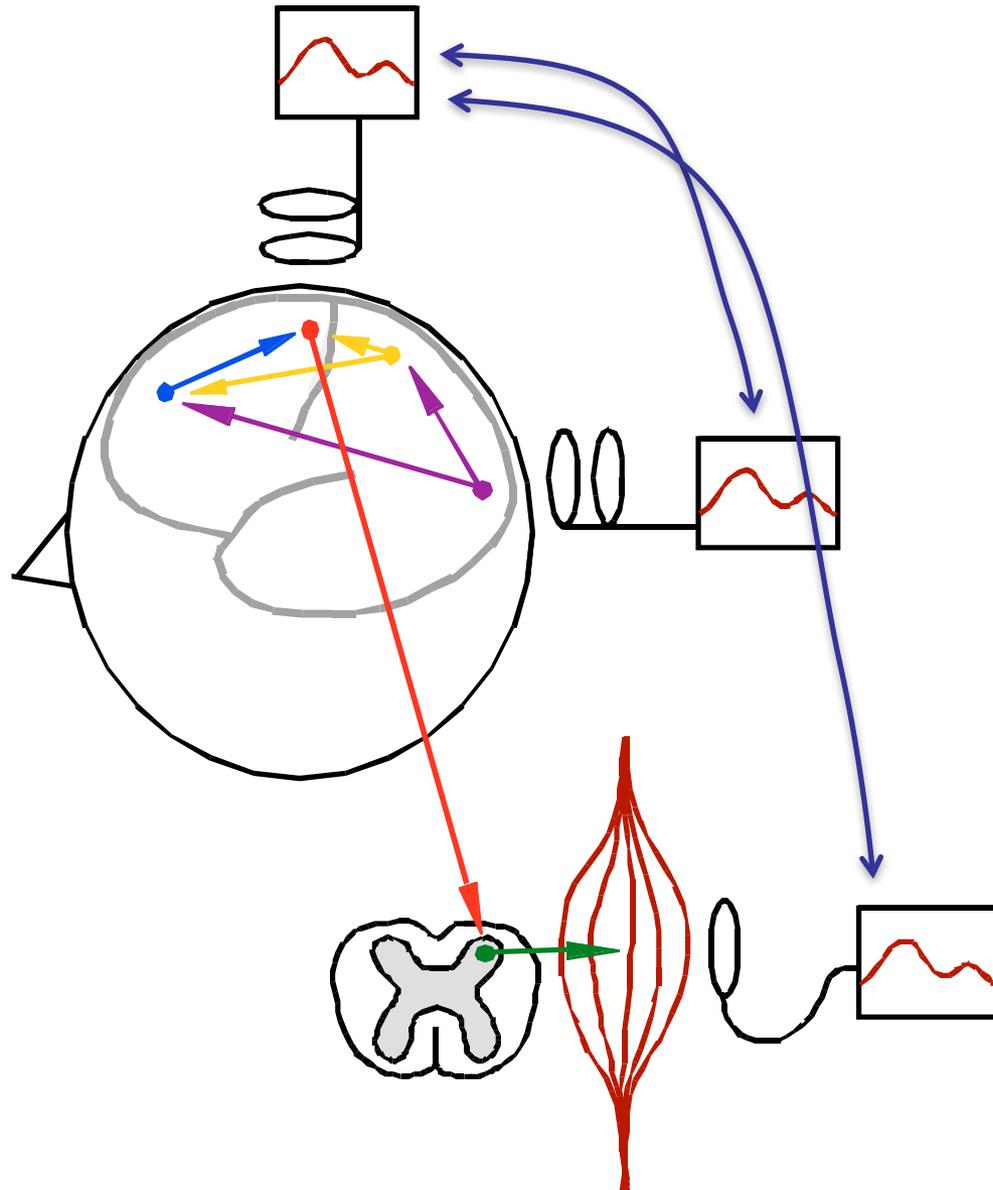
```
graph LR; A["timecourse of activity  
-> ERP  
spectral characteristics  
-> power spectrum  
temporal changes in power  
-> time-frequency response (TFR)"] --> B["source level  
timecourses and  
spectral details"]; C["spatial distribution of activity  
-> source reconstruction"] --> B;
```

source level  
timecourses and  
spectral details

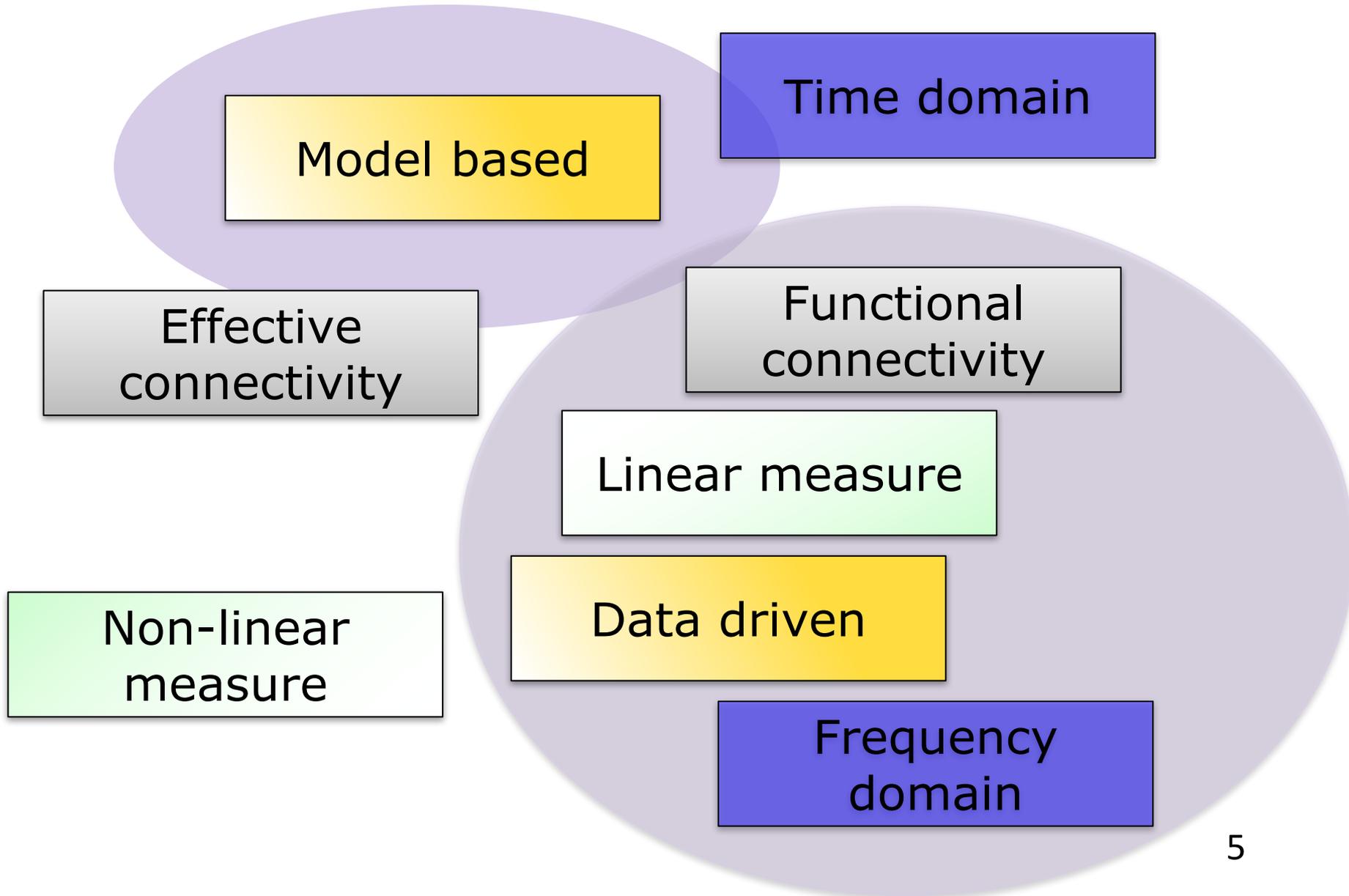
# Univariate analysis



# Connectivity analysis: Beyond univariate analysis



# Measures of connectivity



# Measures of frequency domain connectivity

Coherence coefficient

Phase lag index

Phase synchronization

Partial directed coherence

Synchronization likelihood



Directed transfer function

Phase locking value

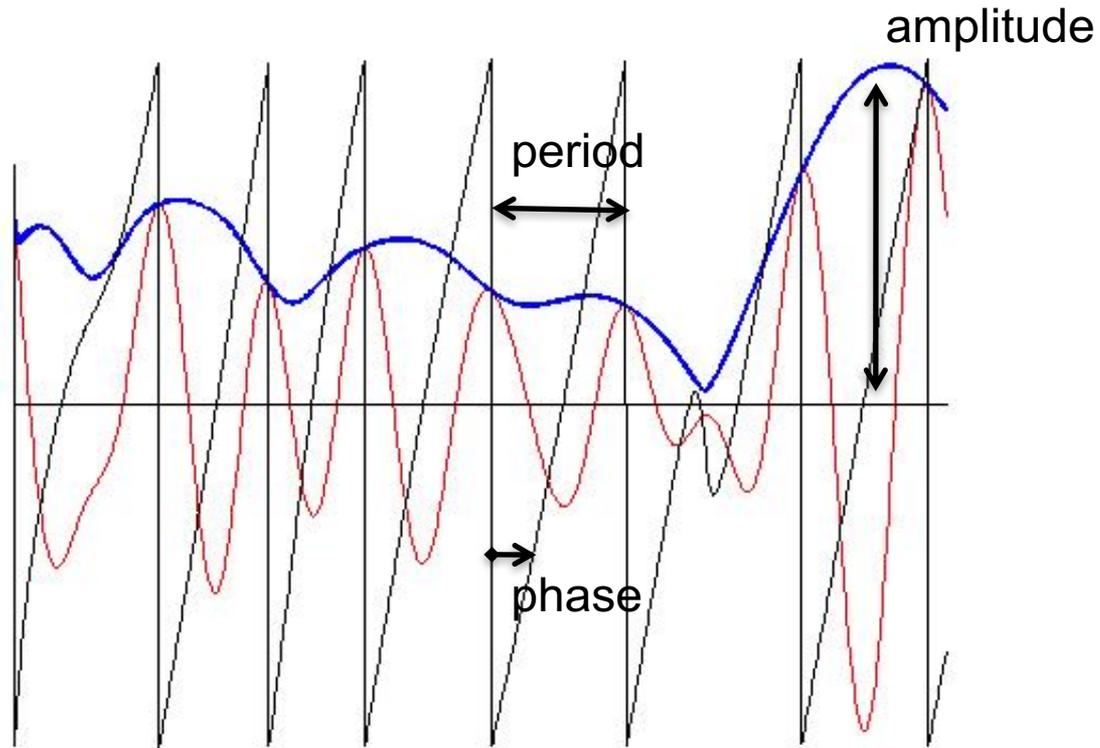
Imaginary part of coherency

Pairwise phase consistency

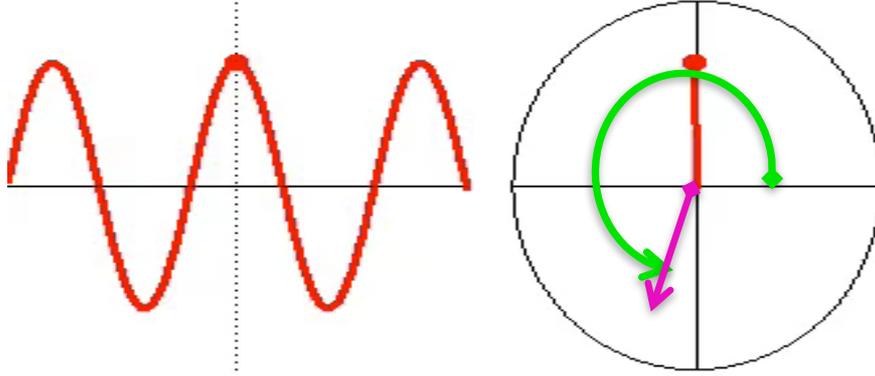
Phase slope index

Frequency domain granger causality

# What constitutes an oscillation? (recap)



# What constitutes an oscillation? (the movie)



$$x = A e^{i\phi}$$

What about 2 oscillations?

Let's look at the phase difference

phase signal 1

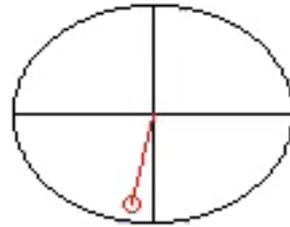
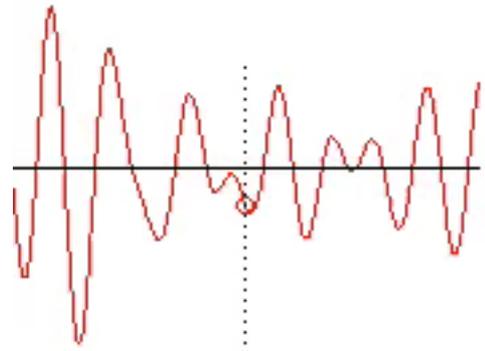
phase difference

phase signal 2

Phase difference is scattered:  
Low synchrony

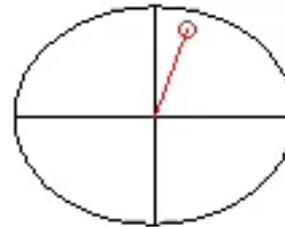
What about 2 oscillations?

Let's look at the phase difference

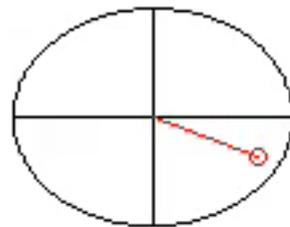
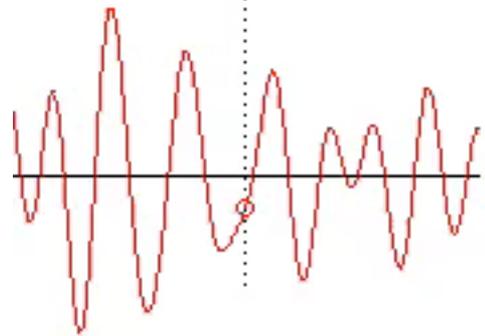


$$x_1 = A_1 e^{i\phi_1}$$

phase signal 1



phase difference



$$x_2 = A_2 e^{i\phi_2}$$

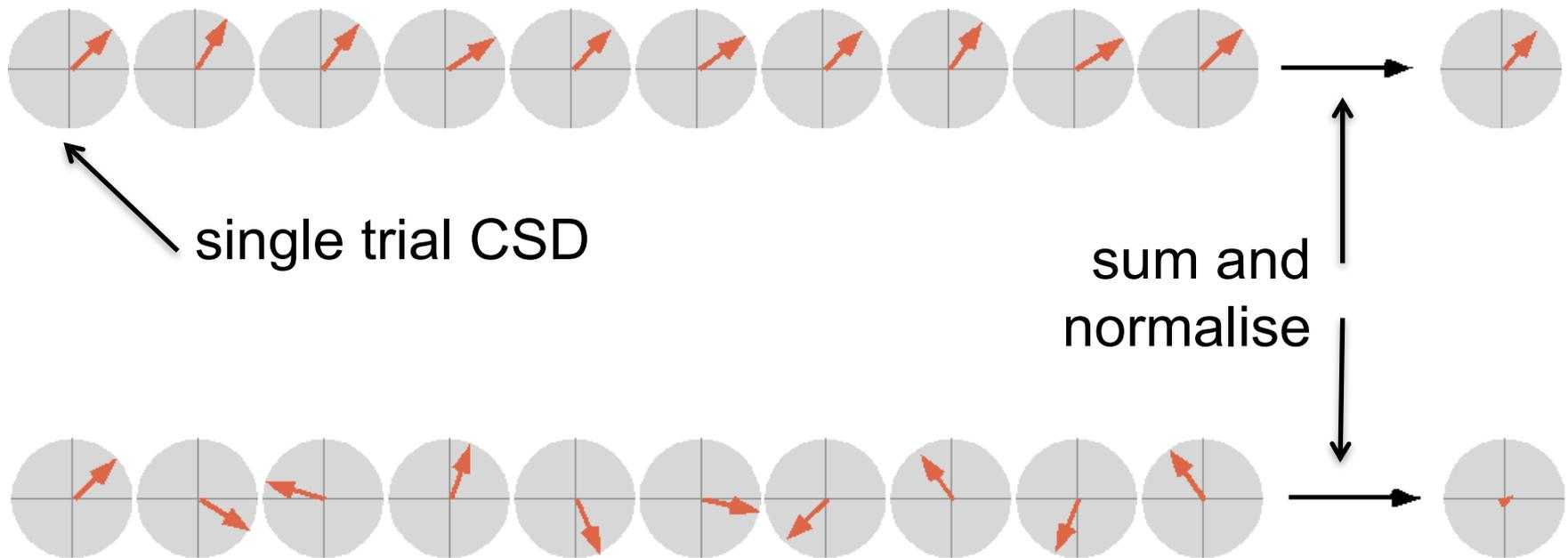
phase signal 2

Phase difference is clustered:  
High synchrony

## Measures of connectivity: coherence (the math view)

Coherence is computed from the *cross-spectral density*, which is obtained by *conjugate multiplication* of the frequency domain representation of the signals

$$x_1 x_2^* = A_1 e^{i\varphi_1} \times A_2 e^{-i\varphi_2} = A_1 A_2 e^{i(\varphi_1 - \varphi_2)}$$



## Measures of connectivity: coherence & co

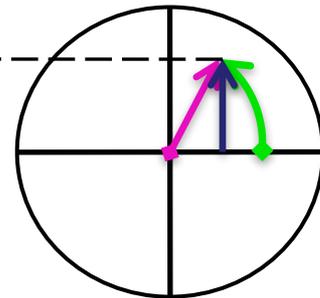
$$\text{Coherence} = \left| \frac{1/N \sum A_1 A_2 e^{i(\varphi_1 - \varphi_2)}}{\sqrt{(1/N \sum A_1^2)(1/N \sum A_2^2)}} \right|$$

$$\text{PLV} = \left| \frac{1/N \sum 1_x 1_x e^{i(\varphi_1 - \varphi_2)}}{\sqrt{(1/N \sum 1^2)(1/N \sum 1^2)}} \right| = \left| \frac{\sum e^{i(\varphi_1 - \varphi_2)}}{N} \right|$$

# Measures of connectivity: coherence & co

$$\text{Coherency} = \frac{1/N \sum A_1 A_2 e^{i(\phi_1 - \phi_2)}}{\sqrt{(1/N \sum A_1^2)(1/N \sum A_2^2)}} = C e^{i\Delta\phi}$$

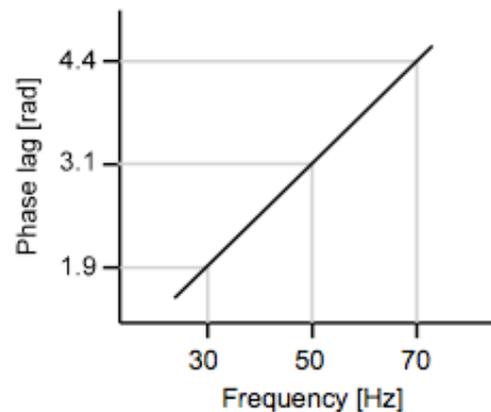
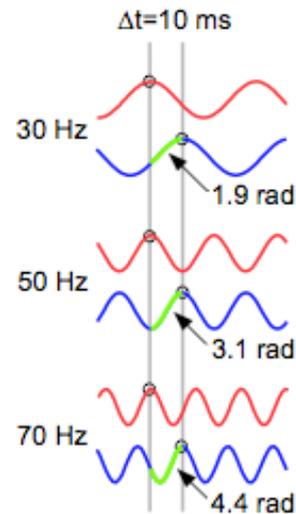
Imaginary part of coherency



# Measures of connectivity: coherence & co

$$\text{Coherency} = \frac{1/N \sum A_1 A_2 e^{i(\phi_1 - \phi_2)}}{\sqrt{(1/N \sum A_1^2)(1/N \sum A_2^2)}} = C e^{i\Delta\phi}$$

Slope of relative phase spectrum indicates time delay



# Coherence and linear prediction

Coherence coefficient  $\sim$  cross-correlation coefficient

$|\text{Coherence}|^2 \sim$  % variance explained

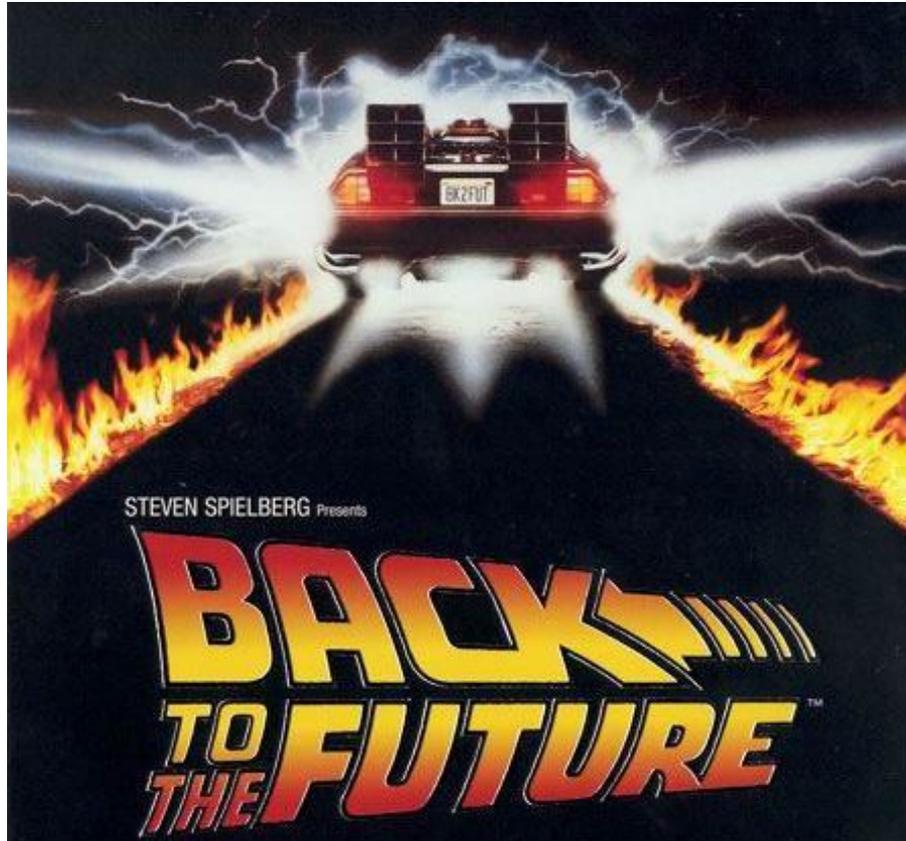
Coherence coefficient similar to frequency domain regression

Conceptual difference with regression: independent and dependent variable are interchangeable

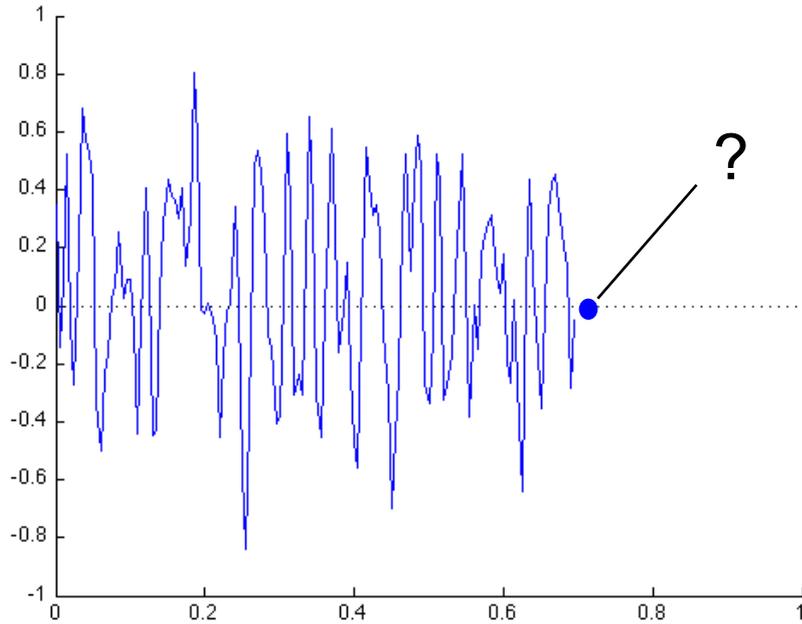
Slope of relative phase spectrum indicates the temporal precedence ( $\sim$  directed influence)

Slope often hard to estimate or close to zero

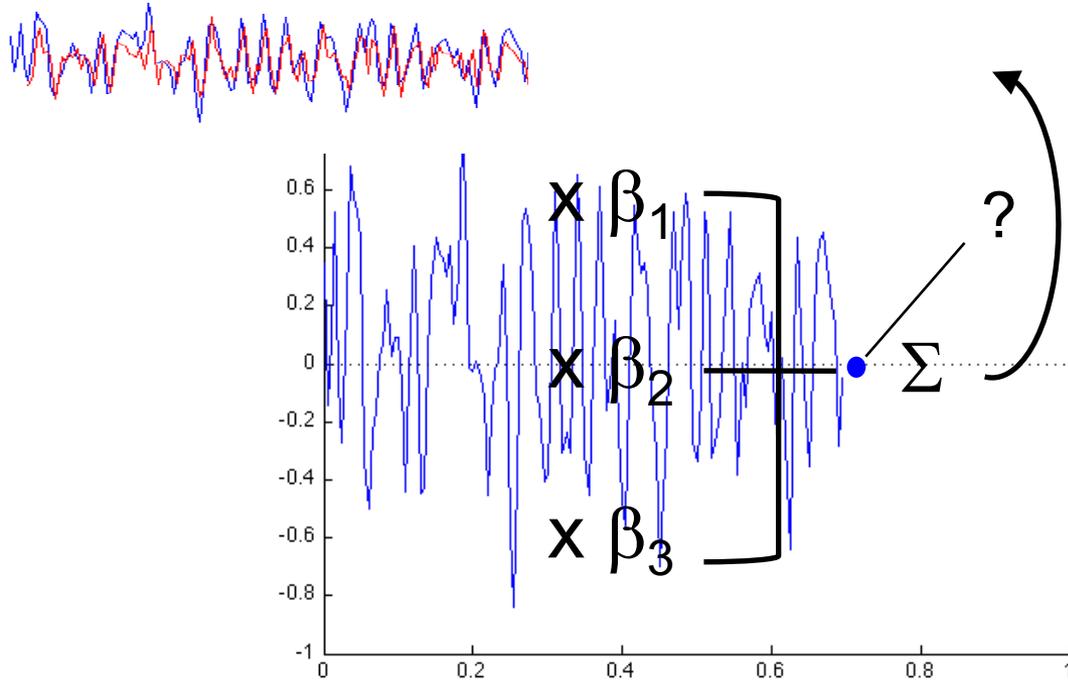
# Linear prediction and directed interaction: the concept of Granger causality



# Linear prediction and directed interaction: the concept of Granger causality



# Linear prediction: autoregressive models



$$X(t) = \sum \beta_{\tau} X(t-\tau) + \eta$$

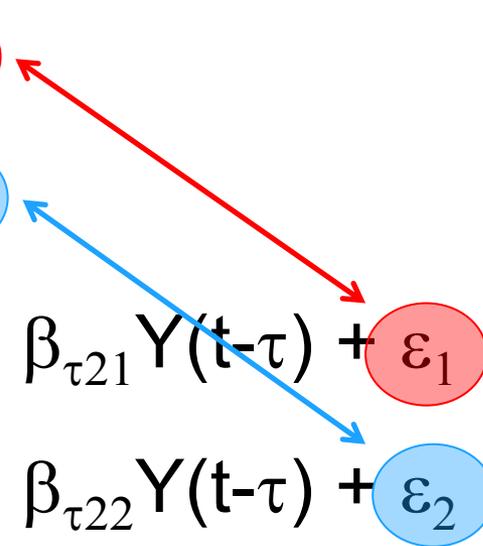
## Two signals: bivariate autoregressive models

$$X(t) = \sum \beta_{\tau_1} X(t-\tau) + \eta_1$$

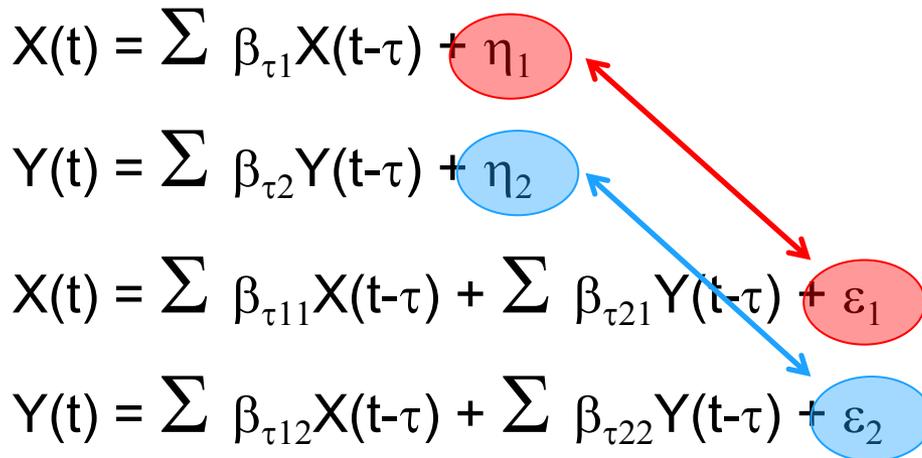
$$Y(t) = \sum \beta_{\tau_2} Y(t-\tau) + \eta_2$$

$$X(t) = \sum \beta_{\tau_{11}} X(t-\tau) + \sum \beta_{\tau_{21}} Y(t-\tau) + \varepsilon_1$$

$$Y(t) = \sum \beta_{\tau_{12}} X(t-\tau) + \sum \beta_{\tau_{22}} Y(t-\tau) + \varepsilon_2$$



# Granger causality: compare the residuals

$$\begin{aligned} X(t) &= \sum \beta_{\tau_1} X(t-\tau) + \eta_1 \\ Y(t) &= \sum \beta_{\tau_2} Y(t-\tau) + \eta_2 \\ X(t) &= \sum \beta_{\tau_{11}} X(t-\tau) + \sum \beta_{\tau_{21}} Y(t-\tau) + \varepsilon_1 \\ Y(t) &= \sum \beta_{\tau_{12}} X(t-\tau) + \sum \beta_{\tau_{22}} Y(t-\tau) + \varepsilon_2 \end{aligned}$$


$$F_{Y \rightarrow X} = \ln\left(\frac{\text{var}(\eta_1)}{\text{var}(\varepsilon_1)}\right)$$

$$F_{X \rightarrow Y} = \ln\left(\frac{\text{var}(\eta_2)}{\text{var}(\varepsilon_2)}\right)$$

# Analogy between Granger and 'plain' regression

$$X(t) = \sum \beta_{\tau 1} X(t-\tau) + \eta_1$$

$$Y(t) = \sum \beta_{\tau 2} Y(t-\tau) + \eta_2$$

$$X(t) = \sum \beta_{\tau 11} X(t-\tau) + \sum \beta_{\tau 21} Y(t-\tau) + \varepsilon_1$$

$$Y(t) = \sum \beta_{\tau 12} X(t-\tau) + \sum \beta_{\tau 22} Y(t-\tau) + \varepsilon_2$$

$$\text{data} = \sum \beta_{\kappa} X_{\kappa} + \eta$$

$$\text{data} = \sum \beta'_{\kappa} X_{\kappa} + \beta'_{\kappa+1} X_{\kappa+1} + \varepsilon$$

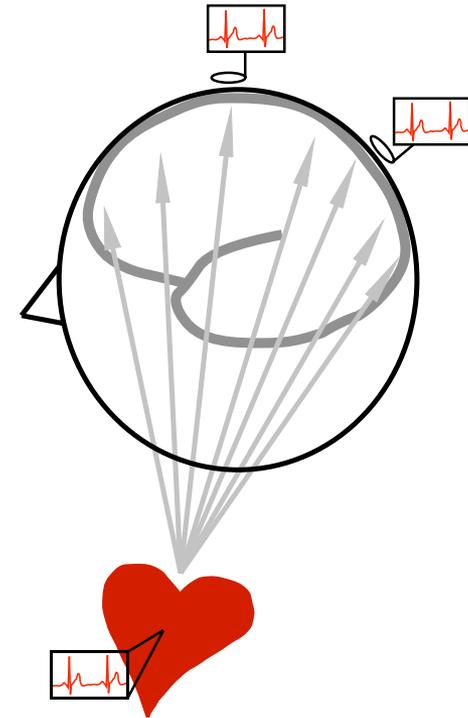
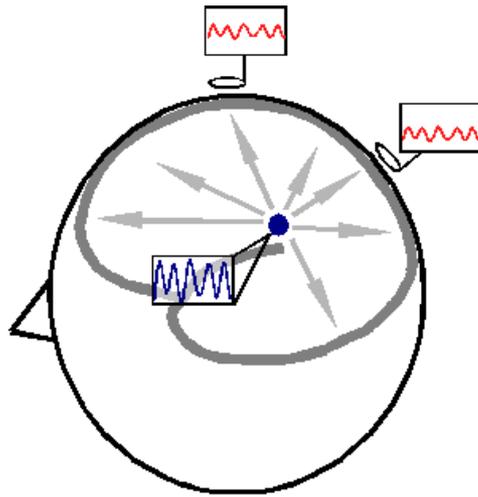
$$F_{Y \rightarrow X} = \ln \left( \frac{\text{var}(\eta_1)}{\text{var}(\varepsilon_1)} \right)$$

$$F \sim \frac{\text{var}(\eta)}{\text{var}(\varepsilon)}$$

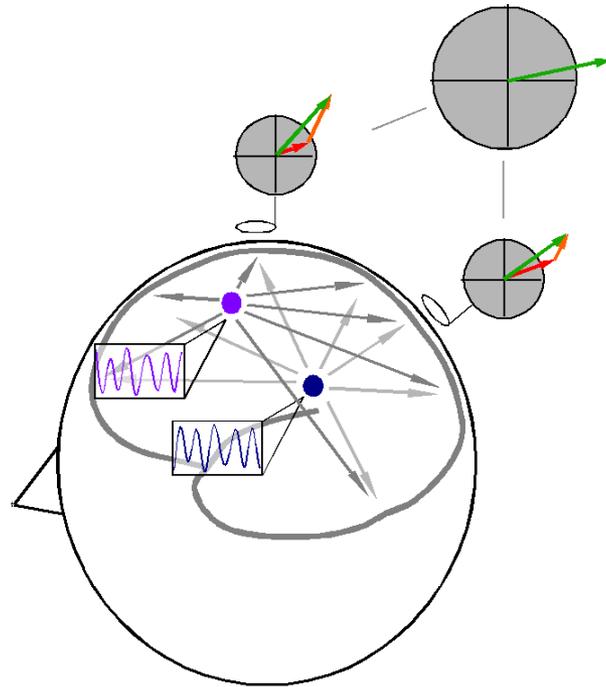
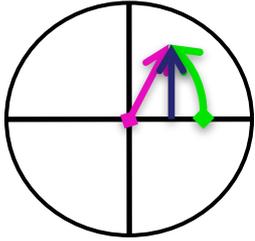
**...only the inference is different**

# Interpretational issues

# Practical issues: Electromagnetic field spread



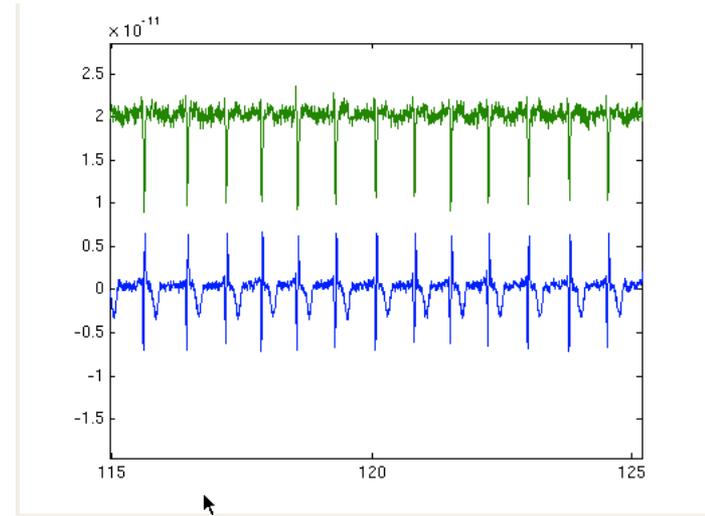
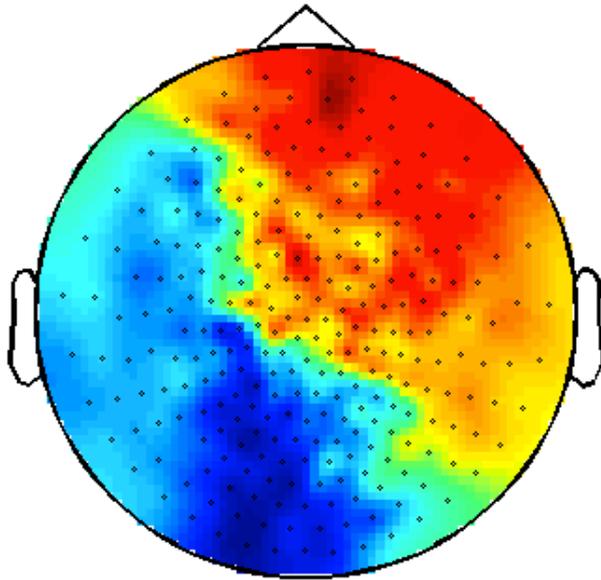
# Practical issues: imaginary part of coherency



$\text{Im}(\text{coherency}) \neq 0$

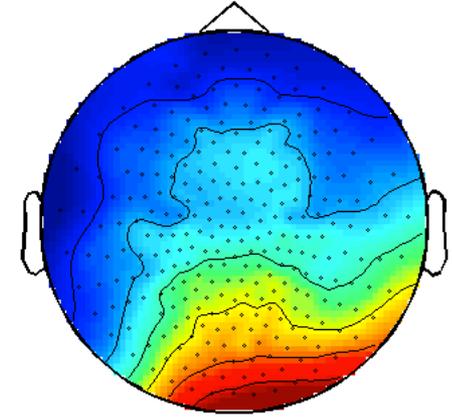
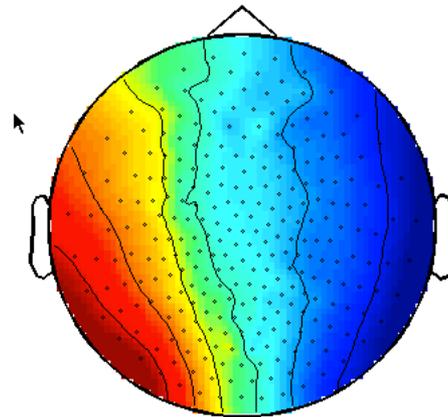
# MEG connectivity: pitfalls with assumptions

WPLI suggests fronto-occipital directed interaction (alpha band)

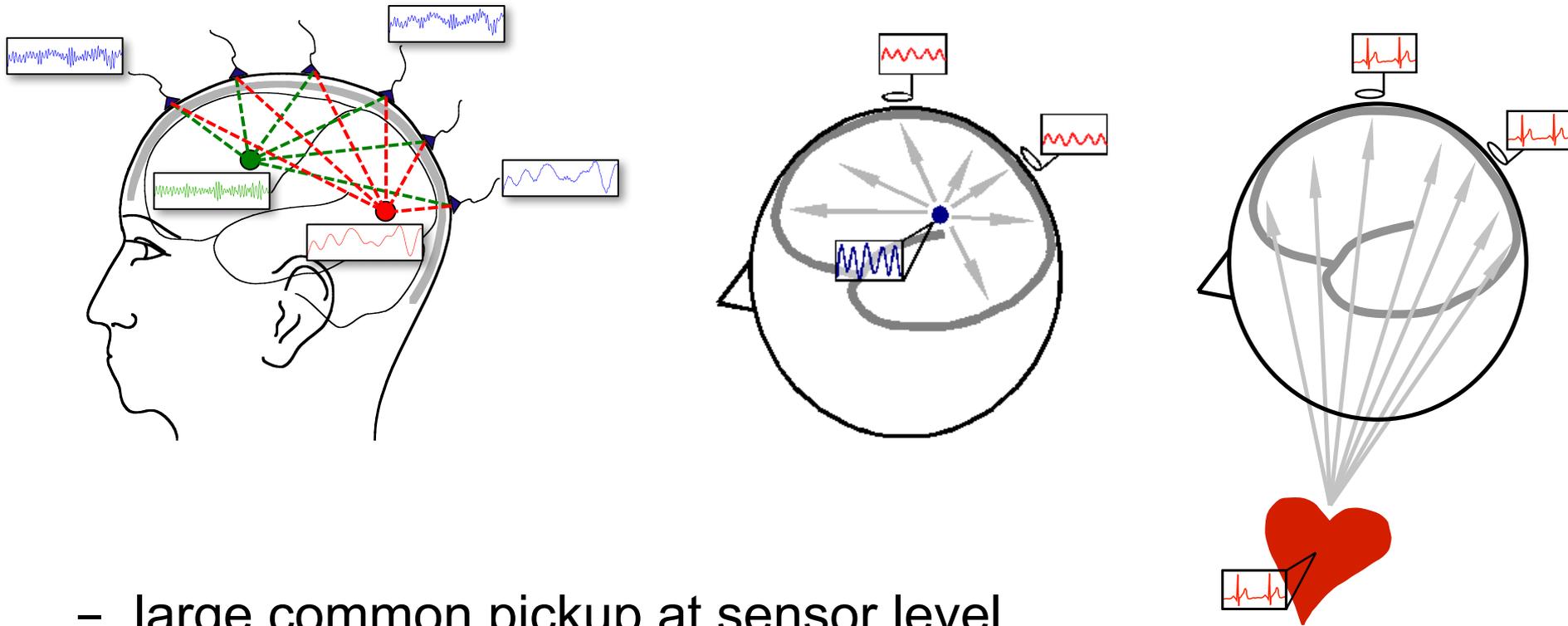


component 1

component 2

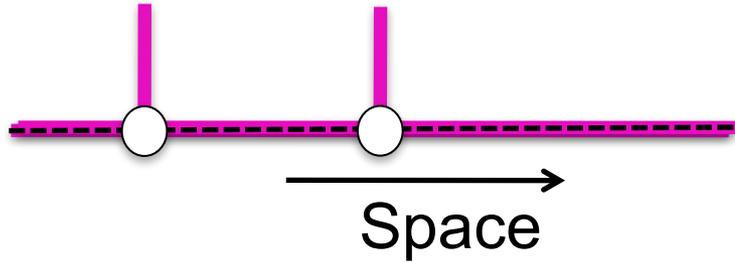


# Common pick up

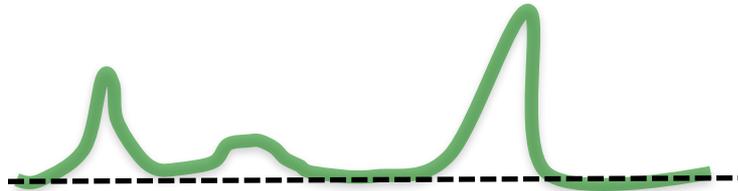


- large common pickup at sensor level
- not all interfering sources are 1-dimensional
- no common pickup if you have a perfect source model
- some common pickup if source model is not perfect

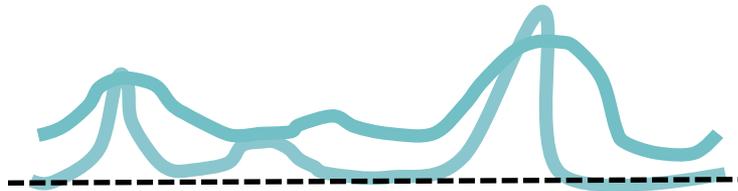
# Features of spatial filters



True source activity

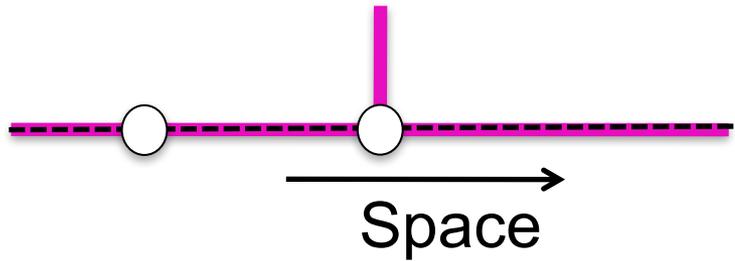


Estimated source activity



# Features of spatial filters:

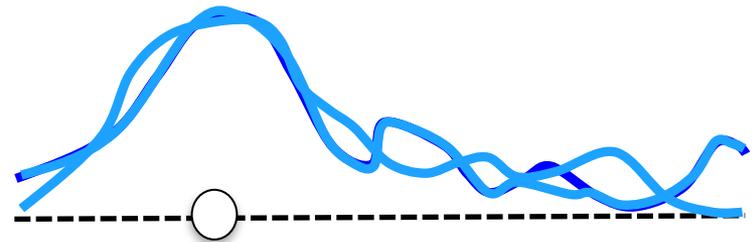
spurious connectivity due to spatial leakage of 'noise'



True source connectivity



Estimated source conn



# Confounds for connectivity

## Common pick up

- other sources in the brain
- other physiological sources
- especially problematic if those sources have some “internal synchronization” themselves

## Differences in signal (or noise) between experimental conditions

- better SNR -> more reliable estimate of the phase
- more reliable phase -> more consistent phase difference

## Concluding remarks

Connectivity analysis is cool

Many measures on the market

Many of the confounds are not easy to deal with

Interpretation of results should therefore  
be done with care